

ELEVENTH EDITION

CALCULUS

AND ITS APPLICATIONS

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calculus

AND ITS APPLICATIONS

ELEVENTH EDITION

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To Elaine, Victoria, and Beth

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Calculus and Its Applications: Expanded Version (ISBN: 978-0-134-12349-3)	
Chapter 7	Trigonometric Functions
Chapter 8	Differential Equations
Chapter 9	Sequences and Series
Chapter 10	Probability Distributions
Chapter 11	(online only) Systems and Matrices
Chapter 12	(online only) Discrete Probability

Preface

Calculus and Its Applications strives to be the most student-oriented applied calculus text on the market, and this eleventh edition continues to improve in that regard. The authors believe that appealing to students' intuition and speaking in a direct, down-to-earth manner make this text accessible to any student possessing the prerequisite math skills. By presenting more topics in a conceptual and often visual manner and adding student self-assessment and teaching aids, this revision addresses students' needs better than ever before. Tapping into areas of student interest, the authors provide motivation through an abundant supply of examples and exercises rich in real-world data from business, economics, environmental studies, health care, and the life sciences.

New examples cover applications ranging from monthly cell phone traffic (in exabytes) to the growth of Facebook membership. Found in every chapter, realistic applications draw students into the discipline and help them to generalize the material and apply it to new situations. To further spark student interest, hundreds of meticulously rendered graphs and illustrations appear throughout the text, making it a favorite among students who are visual learners.

Appropriate for a one-term course, this text is an introduction to applied calculus. A course in intermediate algebra is a prerequisite, although "Appendix A: Review of Basic Algebra," together with "Chapter R: Functions, Graphs, and Models," provides a sufficient foundation to unify the diverse backgrounds of most students. For schools offering a two-term course, *Calculus and Its Applications, Expanded Version*, by the same author team, is available; it includes chapters covering trigonometric functions, differential equations, sequences and series, and probability distributions.

New to This Edition

With every revision comes the opportunity to improve the presentation of the content in a variety of ways based on feedback from users of the text and author experience. This revision is focused on providing students with *just-in-time* reminders of skills that should be familiar to them, updating and adding new applications—especially business applications—throughout the text, and fine-tuning the pedagogical design elements that make this book so user-friendly. The exercises have been carefully evaluated to be sure they align with the objectives of the text, and data analysis of MyMathLab exercises has helped guide the gradation and organization of the exercises in both the text and MyMathLab. Throughout the preface, the new elements of the text are described and specific new and modified content outlined. Below is a quick snapshot of what's new in this edition.

- Additional coverage of exponential functions has been added to Section R.5, to support students' use of these functions in applications.
- The former Section 3.6, "An Economics Application: Elasticity of Demand," has been moved and is now Section 2.7, allowing students to be introduced to the application sooner so they can use the concept throughout Chapter 3.
- A discussion of annuities has been added to Section 3.5, and a new section dedicated to amortization (with the use of Excel as an option) has been added as Section 3.6.
- Section 5.6, "Volume," now includes discussion on the use of shells, and Section 6.4, "An Application: The Least-Squares Technique," was expanded to include a derivation of an exponential regression by hand.
- Applications have been updated to reflect more current data and trends whenever possible.

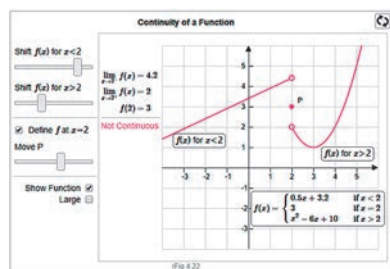
- Exercise sets have been carefully evaluated to ensure appropriate gradation of level, odd/even pairing, and specific connection to the objectives being evaluated. In addition, MyMathLab usage data was analyzed to expose any exercises that needed improvement.
- MyMathLab has been greatly improved to include a vast array of new resources for both instructors and students. In addition to a greater quantity and variety of exercises, MyMathLab now includes more videos, additional levels of assessment, and interactive figures to help students gain the skills and knowledge they need to be successful in this and future courses.

Our Approach

Intuitive Presentation

Although the word *intuitive* has many meanings and interpretations, its use here means “experience based, without proof.” Throughout the text, when a concept is discussed, its presentation is designed so that the students’ learning process is based on their earlier mathematical experience. This is illustrated by the following situations.

- Before the formal definition of *continuity* is presented, an informal explanation is given, complete with graphs that make use of student intuition about ways in which a function could be discontinuous (see pp. 109–110).
- The definition of *derivative*, in Chapter 1, is presented in the context of a discussion of average rates of change (see p. 129). This presentation is more accessible and realistic than the strictly geometric idea of slope.
- When maximization problems involving volume are introduced (see pp. 244–246), a function is derived that is to be maximized. Instead of forging ahead with the standard calculus solution, the text first asks the student to make a table of function values, graph the function, and then estimate the maximum value. This experience provides students with more insight into the problem. They recognize not only that different dimensions yield different volumes, but also that the dimensions yielding the maximum volume may be conjectured or estimated as a result of the calculations.
- The explanation underlying the definition of the number e is presented in Chapter 3 both graphically and through a discussion of continuously compounded interest (see pp. 330–331).
- Within MyMathLab, students and instructors have access to **interactive figures** that illustrate concepts and allow manipulation by the user so that he or she can better predict and understand the underlying concepts. Also available within MyMathLab are questions that provide focus for student use.



Timely Help for Gaps in Algebra Skills

One of the most critical factors underlying success in this course is a strong foundation in algebra skills. We recognize that students start this course with varying degrees of skills, so we have included multiple opportunities in both the text and MyMathLab to help students target their weak areas and remediate or refresh the needed skills.

In the text

- **Prerequisite Skills Diagnostic Test (Part A)**. This portion of the diagnostic test assesses skills refreshed in Appendix A: Review of Basic Algebra. Answers to the questions reference specific examples within the appendix.
- **Appendix A: Review of Basic Algebra**. This 11-page appendix provides examples on topics such as exponents, equations, and inequalities and applied problems. It ends with an exercise set, for which answers are provided at the back of the book so students can check their understanding.
- **Prerequisite Skills Diagnostic Test (Part B)**. This portion of the diagnostic test assesses skills that are reviewed in “Chapter R: Functions, Graphs, and Models,” and the answers reference specific sections in that chapter. Some instructors may choose

to cover these topics thoroughly in class, making this assessment less critical. Other instructors may use all or portions of this test to determine whether there is a need to spend time remediating before moving on with Chapter 1.

- **Chapter R: Functions, Graphs, and Models.** This chapter covers basic concepts related to functions, graphing, and modeling. It is an optional chapter based on students' prerequisite skills.

In MyMathLab

- **Integrated Review.** You can diagnose weak prerequisite skills through built-in diagnostic quizzes. By coupling these quizzes with Personalized Homework, MyMathLab provides remediation for just those skills a student lacks. Even if you choose not to assign these quizzes with Personalized Homework, students can self-remediate through videos and practice exercises provided at the objective level. MyMathLab provides the just-in-time help that students need, so you can focus on the course content.
- **New! Basic Skills Videos.** Videos are now available within exercises to help refresh the key algebra skill required for a specific exercise.

Exercises and Applications

There are over 3500 assignable section-level homework exercises in this edition. A large percentage of these exercises are rendered algorithmically in MyMathLab. All exercise sets are enhanced by the inclusion of real-world applications, detailed art pieces, and illustrative graphs. There are a variety of types of exercises, too, so different levels of understanding and varying approaches to problems can be assessed. In addition to applications, the exercise sets include *Thinking and Writing*, *Synthesis*, *Technology Connection*, and *Concept Reinforcement* exercises. The exercises in MyMathLab reflect the depth and variety of those in the printed text.

The authors also provide *Quick Check* exercises, following selected examples, to give students the opportunity to check their understanding of new concepts or skills as soon as they learn them and one skill at a time. Instructors may include these as part of a lecture as a means of gauging skills and gaining immediate student feedback. Answers to the Quick Check exercises are provided following the exercise set at the end of each section.

Relevant and factual applications drawn from a broad spectrum of fields are integrated throughout the text as applied examples and exercises, and are also featured in separate application sections. Applications have been updated and expanded in this edition to include even more real data. In addition, each chapter opener features an application that serves as a preview of what students will learn in the chapter.

The applications in the exercise sets in the text and within MyMathLab are grouped under headings that identify them as reflecting real-life situations: Business and Economics, Life and Physical Sciences, Social Sciences, and General Interest. This organization allows the instructor to gear the assigned exercises to specific students and also allows each student to know whether a particular exercise applies to his or her major.

Furthermore, the Index of Applications at the back of the book provides students and instructors with a comprehensive list of the many different fields considered throughout the text.

Opportunities to Incorporate Technology

This edition continues to emphasize mathematical modeling, utilizing the advantages of technology as appropriate. The use of Excel as a tool for solving problems has been expanded in this edition. Though the use of technology is optional with this text, its use meshes well with the text's more intuitive approach to applied calculus.

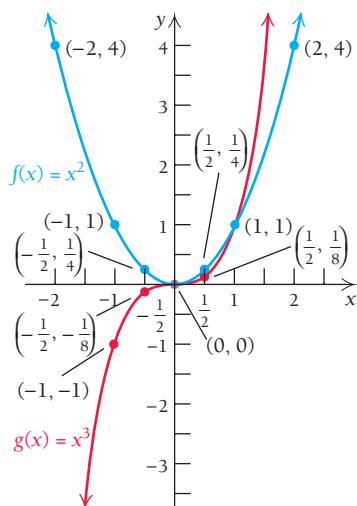
technology connections

Technology Connections are included throughout the text to illustrate the use of technology, including graphing calculators, Excel spreadsheets, and smartphone apps. Whenever appropriate, art that simulates graphs or tables generated by a graphing calculator is included as well. The goal is to take advantage of technology to which many students have access, wherever it makes sense, given the mathematical situation.

Four types of Technology Connections allow students and instructors to explore key ideas:

- **Lesson/Teaching.** These provide students with an example, followed by exercises to work within the lesson.
- **Checking.** These tell the students how to verify a solution within an example by using a graphing calculator.
- **Exploratory/Investigation.** These provide questions to guide students through an investigation.
- **Technology Connection Exercises.** Most exercise sets contain technology-based exercises identified with either an icon or the heading “Technology Connection.” This type of exercise also appears in the Chapter Review Exercises, Chapter Tests, and the supplemental Printable Test Forms.

Extended Technology Applications at the end of every chapter use real applications and real data. They require a step-by-step analysis that encourages group work. More challenging in nature, the exercises in these features often involve the use of regression to create models on a graphing calculator. The data in the Extended Technology Applications has been updated wherever possible to keep the applications fresh for instructors and relevant for students.



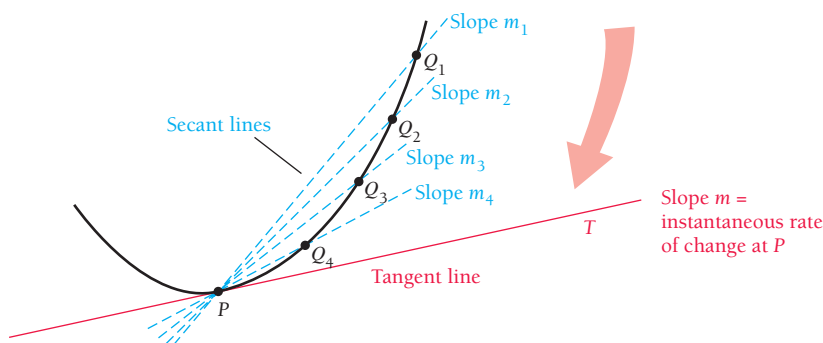
Use of Art and Color

One of the hallmarks of this text is the pervasive use of color as a pedagogical tool. Color is used in a methodical and precise manner that enhances the readability of the text for students and instructors.

- When two curves are graphed using the same set of axes, one is usually red and the other blue, with the red graph being the curve of major importance. The equation labels are the same color as the curve for clarity (see p. 52).
- When the instructions say “Graph,” the dots match the color of the curve. When dots are used for emphasis other than just merely plotting, they are black.

Throughout the text, blue is used for secant lines and red for tangent lines (see p. 128).

- Red denotes substitution in equations while blue highlights the corresponding outputs, and the specific use of color is carried out in related figures (see pp. 206–207).
- Beginning with the discussion of integration, an amber color is used to highlight areas in graphs (see p. 400).



Opportunity for Review and Synthesis

Recognizing that it is often while preparing for exams that concepts gel for students, this text offers abundant opportunities for students to review, analyze, and synthesize recently learned concepts and skills.

- A **Section Summary** precedes every exercise set to assist students in identifying the key topics for each section and to serve as a mini-review.
- A **Chapter Summary** at the end of every chapter includes a section-by-section list of key definitions, concepts, and theorems, with examples for further clarification.
- **Chapter Review Exercises**, which include bracketed references to the section in which the related concept is first introduced, provide comprehensive coverage and appropriate referencing of each chapter's material.
- A **Chapter Test** at the end of every chapter and a **Cumulative Review** at the end of the text give students an authentic exam-like environment for testing their mastery. Answers to the chapter tests and the Cumulative Review are at the back of the text and include section references so students can diagnose their mistakes while preparing for exams.

New and Revised Content

In response to faculty and student feedback, we have made many changes to the text's content for this edition. New examples and exercises have been added throughout each chapter, as well as new problems to each chapter's review exercises and chapter test. Data have been updated wherever achievable, so problems use the most up-to-date information possible. Following is an overview of the major content changes in each chapter.

chapter r

Several new exercises have been added to this chapter, including 10 in Section R.2 that address how to describe functions verbally and translate them algebraically. In Section R.5, a subsection and several exercises were added that cover exponential functions and their graphs to help bridge the gap in student understanding of those functions. Section R.6 has expanded discussion of exponential functions, along with four new exercises involving exponential models. Throughout, data-driven examples were updated when possible.

chapter 1

The goal for Chapter 1 was to update the data-based examples and consolidate similar exercises into a more manageable number. Section 1.8 has changed the most, with the addition of examples and exercises designed to help students visualize acceleration and velocity. In addition, l'Hôpital's Rule is briefly covered in a synthesis exercise.

chapter 2

In Section 2.5, Example 3 has been rewritten to factor in cost, spreadsheet use has been added to show how a minimum or maximum can be found numerically, a new Technology Connection has been added, and Examples 6 and 7 were integrated into a single example. Several examples in Section 2.6 have been consolidated, and a new Quick Check exercise has been added. The main change to Chapter 2 is the addition of the expanded and updated Section 2.7, "Elasticity." This new location is a more natural fit than its former position as Section 3.6. The former Section 2.7, "Implicit Differentiation and Related Rates," has become Section 2.8.

chapter 3

New material on exponential functions has been added to Section 3.1 based on the expanded content in Section R.5. In Section 3.2, there is more emphasis on the general anti-derivative for $1/x$, for all x except $x = 0$, through additional examples and exercises. The Rule of 70 is now included in Section 3.3, which also has a new Technology Connection. Section 3.5 has been expanded to include a discussion on annuities. New Section 3.6 covers the topic of amortization and includes some Excel spreadsheet applications.

chapter 4

Application examples and exercises were added and updated throughout Chapter 4. In addition, new material on Simpson's Rule was added to Section 4.2, and the topic of recursion was moved to a synthesis exercise in Section 4.6.

chapter 5

Topics throughout Chapter 5 were expanded, allowing for over 80 new exercises and applications. Improper integration at a vertical asymptote was added to Section 5.3, and finding volume by shells was added to Section 5.6.

chapter 6

The main changes in Chapter 6 are the addition of exponential regression to Section 6.4 and the addition of average value of a two-variable function to Section 6.6. New application exercises were also added, and data-driven examples and exercises were updated throughout.

Appendix A

New material on the Principle of Square Roots was added as a reference for students.

Supplements

STUDENT SUPPLEMENTS

Student's Solutions Manual

(ISBN: 0-321-99905-3/978-0-321-99905-4)

- Provides detailed solutions to all odd-numbered section exercises, with the exception of the Thinking and Writing exercises

Graphing Calculator Manual for Applied Mathematics (downloadable)

- By Victoria Baker, Nicholls State University
- Contains detailed instruction for using the TI-83/TI-83+/TI-84+C
- Instructions are organized by topic.
- Downloadable from within MyMathLab

Excel Spreadsheet Manual for Applied Mathematics (downloadable)

- By Stela Pudar-Hozo, Indiana University-Northwest
- Contains detailed instruction for using Excel 2013
- Instructions are organized by topic.
- Downloadable from within MyMathLab

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- Available to qualified instructors through the Pearson Instructor Resource Center, www.pearsonhighered.com/irc, and MyMathLab

Printable Test Forms (downloadable)

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- Contains four comprehensive final exams
- Includes answer keys
- Available to qualified instructors through the Pearson Instructor Resource Center, www.pearsonhighered.com/irc, and MyMathLab

PowerPoint Lecture Presentations

- Classroom presentation software oriented specifically to the text's topic sequence
- Available to qualified instructors through the Pearson Instructor Resource Center, www.pearsonhighered.com/irc, and MyMathLab

TECHNOLOGY RESOURCES

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 - The results of the chapter skills check quiz can be tied to personalized homework so that each student is given targeted skills to practice prior to starting course-level work.
- **Exercises:** The homework and practice exercises in MyMathLab are correlated to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunity for practice and mastery. The software offers immediate, helpful feedback when students enter incorrect answers.
- **Multimedia Learning Aids:** Exercises include guided solutions, sample problems, animations, videos, and eText access for extra help at point-of-use.
 - Videos include section-level, lecture-style videos as well as example-level videos. Both are helpful resources when instructors or tutors are not available.
 - *MathTalk* videos show how the math the students are learning now may apply to their future careers. These videos could be used to kick off a lecture, in a flipped classroom setting, or as a way to motivate students.
 - Videos can be assigned as homework or as a prerequisite to students getting started on the homework. Pre-made questions are available for the MathTalk videos.
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Prerequisite skills diagnostic test

To the Student and the Instructor

Part A of this diagnostic test covers basic algebra concepts, such as properties of exponents, multiplying and factoring polynomials, equation solving, and applied problems. Part B covers topics, discussed in Chapter R, such as graphs, slope, equations of lines, and functions, most of which come from a course in intermediate or college algebra. This diagnostic test does not cover regression, though it is considered in Chapter R and used throughout the text. This test can be used to assess student needs for this course. Students who miss most of the questions in part A should study Appendix A before moving to Chapter R. Those who miss most of the questions in part B should study Chapter R. Students who miss just a few questions might study the related topics in either Appendix A or Chapter R before continuing with the calculus chapters.

Part A

Answers and locations of worked-out solutions appear on p. A-39.

Express each of the following without an exponent.

1. 4^3 2. $(-2)^5$ 3. $(\frac{1}{2})^3$ 4. $(-2x)^1$ 5. e^0

Express each of the following without a negative exponent.

6. x^{-5} 7. $(\frac{1}{4})^{-2}$ 8. t^{-1}

Multiply. Express each answer without a negative exponent.

9. $x^5 \cdot x^6$ 10. $x^{-5} \cdot x^6$ 11. $2x^{-3} \cdot 5x^{-4}$

Divide. Express each answer without a negative exponent.

12. $\frac{a^3}{a^2}$ 13. $\frac{e^3}{e^{-4}}$

Simplify. Express each answer without a negative exponent.

14. $(x^{-2})^3$ 15. $(2x^4y^{-5}z^3)^{-3}$

Multiply.

16. $3(x - 5)$ 17. $(x - 5)(x + 3)$

18. $(a + b)(a + b)$

19. $(2x - t)^2$ 20. $(3c + d)(3c - d)$

Factor.

21. $2xh + h^2$ 22. $x^2 - 6xy + 9y^2$

23. $x^2 - 5x - 14$ 24. $6x^2 + 7x - 5$

25. $x^3 - 7x^2 - 4x + 28$

Solve.

26. $-\frac{5}{6}x + 10 = \frac{1}{2}x + 2$ 27. $3x(x - 2)(5x + 4) = 0$

28. $4x^3 = x$ 29. $\frac{2x}{x - 3} - \frac{6}{x} = \frac{18}{x^2 - 3x}$

30. $17 - 8x \geq 5x - 4$

31. After a 5% gain in weight, a grizzly bear weighs 693 lb. What was the bear's original weight?

32. Raggs, Ltd., a clothing firm, determines that its total revenue, in dollars, from the sale of x suits is given by $200x + 50$. Determine the number of suits the firm must sell to ensure that its total revenue will be more than \$70,050.

Part B

Answers and locations of worked-out solutions appear on p. A-39.

Graph.

1. $y = 2x + 1$ 2. $3x + 5y = 10$

3. $y = x^2 - 1$ 4. $x = y^2$

5. A function f is given by $f(x) = 3x^2 - 2x + 8$. Find each of the following: $f(0)$, $f(-5)$, and $f(7a)$.

6. A function f is given by $f(x) = (x + 3)^2 - 4$. Find all x such that $f(x) = 0$.

7. Graph the function f defined as follows:

$$f(x) = \begin{cases} 4, & \text{for } x \leq 0, \\ 3 - x^2, & \text{for } 0 < x \leq 2, \\ 2x - 6, & \text{for } x > 2. \end{cases}$$

8. Write interval notation for $\{x \mid -4 < x \leq 5\}$.

9. Find the domain: $f(x) = \frac{3}{2x - 5}$.

10. Find the slope and y -intercept of $2x - 4y - 7 = 0$.

11. Find an equation of the line that has slope 3 and contains the point $(-1, -5)$.

12. Find the slope of the line containing the points $(-2, 6)$ and $(-4, 9)$.

Graph.

13. $f(x) = x^2 - 2x - 3$ 14. $f(x) = x^3$ 15. $f(x) = \frac{1}{x}$

16. $f(x) = |x|$ 17. $f(x) = -\sqrt{x}$

18. Suppose that \$1000 is invested at 5%, compounded annually. How much is the investment worth at the end of 2 yr?

Credits

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Appendix B pp. 598–599: Screenshots from Microsoft® Excel®. Used by permission of Microsoft Corporation.

R Functions, Graphs, and Models



What You'll Learn

- R.1** Graphs and Equations
- R.2** Functions and Models
- R.3** Finding Domain and Range
- R.4** Slope and Linear Functions
- R.5** Nonlinear Functions and Models
- R.6** Mathematical Modeling and Curve Fitting

Why It's Important

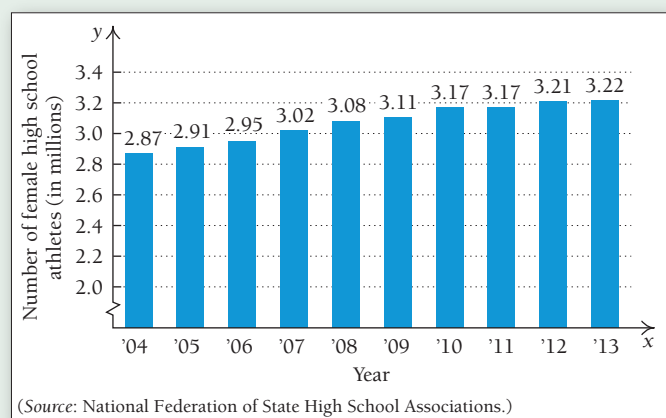
This chapter introduces functions and covers their notation, graphs, and applications. Also presented are many topics considered often throughout the text: supply and demand; total cost, revenue, and profit; the concept of a mathematical model; and curve fitting.

Skills in using a graphing calculator are introduced in optional Technology Connections. Details on keystrokes are given in the *Graphing Calculator Manual* (GCM).

Part A of the diagnostic test (p. xv), on basic algebra concepts, allows students to determine whether they need to review Appendix A (p. 585) before studying this chapter. Part B, on college algebra topics, assesses the need to study this chapter before the calculus chapters.

Where It's Used

Participation of Females in High School Athletics: What is the predicted number of female high school athletes in 2017? (This problem appears as Example 5 in Section R.1.)



R.1

- Graph equations.
- Use graphs as mathematical models to make predictions.
- Carry out calculations involving compound interest.



Graphs and Equations

What Is Calculus?

What is calculus? This is a common question at the start of a course like this. Let's consider a simplified answer for now.

The common food can comes in many sizes, as the photo at the left illustrates. The following is a typical problem from an algebra course. Try to solve it. (If you need some algebra review, refer to Appendix A at the end of the book.)

Algebra Problem

A soup can contains a volume of 250 cm^3 . If the height of the can is twice the length of the radius of the base, find the dimensions of the can.

From geometry, the volume of a cylinder is $V = \pi r^2 h$, where r is the radius of the circular base in centimeters and h is the height in centimeters. Since $V = 250$ and $h = 2r$, we have $\pi r^2(2r) = 250$, or $2\pi r^3 = 250$. Solving for r , we find that the soup can has a radius of approximately 3.4 cm and a height of 6.8 cm.

The following is a calculus problem that a manufacturer of cans might need to solve:

Calculus Problem

A soup can is to contain a volume of 250 cm^3 . If the cost of the material for the two circular ends is \$0.0008 per square centimeter and the cost of the material for the side is \$0.0015 per square centimeter, what dimensions will minimize the cost of the can?

One way to solve this problem might be to choose several sets of dimensions that give a volume for the can of 250 cm^3 , compute the resulting costs, and determine which is the lowest. For any r , we must have $h = 250/\pi r^2$. Using a computer spreadsheet, we could create a table such as the one below. We let r = radius in centimeters and h = height in centimeters.

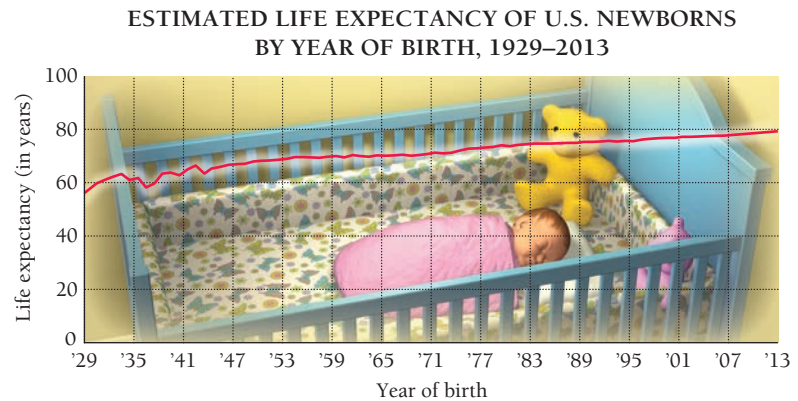
	A	B	C
1			
2	Radius	Height	Cost
3	r	h	C
4	3.5	6.495277933	0.275868914
5	3.6	6.139440947	0.273485845
6	3.7	5.812063892	0.271525071
7	3.8	5.510190767	0.269961189
8	3.9	5.231239624	0.268771404
9	4	4.972947167	0.2679352
10	4.1	4.733322705	0.267434061
11	4.2	4.510609676	0.267251237 ← Possible lowest cost
12	4.3	4.303253363	0.267371533
13	4.4	4.109873692	0.267781137
14	4.5	3.929242206	0.268467467

From the data in the table, we might conclude that the lowest cost is about \$0.26725 per can when the can's radius is 4.2 cm and its height is 4.51 cm. But how can we be certain that no other dimensions will give a lower cost? We need the tools of calculus to answer this. In Chapter 2, we will study such maximum–minimum problems, including a complete solution to this one, which appears as Example 3 in Section 2.5.

Other topics we will consider in calculus are the slope of a curve at a point, rates of change, area under a curve, accumulations of quantities, and some statistical applications.

Graphs

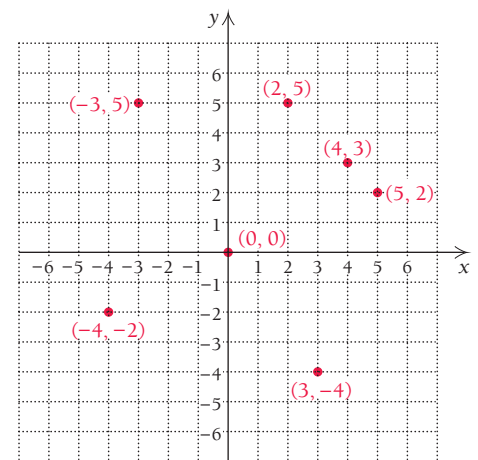
The study of graphs is an essential aspect of calculus. A graph offers the opportunity to visualize relationships. For instance, the graph below shows how life expectancy has changed over time in the United States. One topic that we consider later in calculus is how a change on one axis affects the change on another.



(Source: U.S. National Center for Health Statistics.)

Ordered Pairs and Graphs

Each point in a plane corresponds to an ordered pair of numbers. Note in the figure at the right that the point corresponding to the pair $(2, 5)$ is different from the point corresponding to the pair $(5, 2)$. This is why we call a pair like $(2, 5)$ an *ordered pair*. The first number is called the *first coordinate* of the point, and the second number is called the *second coordinate*. Together these are the *coordinates of the point*. The horizontal line is often labeled as the *x-axis*, and the vertical line is often labeled as the *y-axis*. The two axes intersect at the *origin*, $(0, 0)$.



Graphs of Equations

A **solution** of an equation in two variables is an ordered pair of numbers that, when substituted for the variables, forms a true statement. If not directed otherwise, we usually take the variables in *alphabetical* order. For example, $(-1, 2)$ is a solution of the equation $3x^2 + y = 5$, because when we substitute -1 for x and 2 for y , we get a true statement:

$$\begin{array}{r|l} 3x^2 + y = 5 & \\ \hline 3(-1)^2 + 2 & ? 5 \\ 3 + 2 & | 5 \\ 5 & 5 \text{ TRUE} \end{array}$$

DEFINITION

The **graph** of an equation is a drawing that represents all ordered pairs that are solutions of the equation.

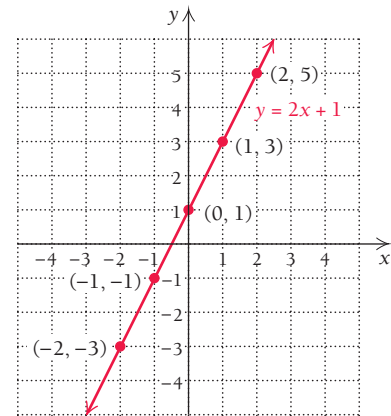
We obtain the graph of an equation by plotting enough ordered pairs (that are solutions) to see a pattern.

EXAMPLE 1 Graph: $y = 2x + 1$.

Solution We first find some ordered pairs that are solutions and arrange them in a table. To find an ordered pair, we can choose any number for x and then determine y . For example, if we choose -2 for x and substitute that value in $y = 2x + 1$, we find that $y = 2(-2) + 1 = -4 + 1 = -3$. Thus, $(-2, -3)$ is a solution. We select both negative numbers and positive numbers, as well as 0, for x .

x	y	(x, y)
-2	-3	$(-2, -3)$
-1	-1	$(-1, -1)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	5	$(2, 5)$

- (1) Choose any x .
- (2) Compute y .
- (3) Form the pair (x, y) .
- (4) Plot the points.

**Quick Check 1** ✓

Graph: $y = 3 - x$.

After we plot the points, we look for a pattern in the graph. In this case, the points suggest a solid line. We draw the line with a straightedge and label it $y = 2x + 1$. 1 ✓

EXAMPLE 2 Graph: $3x + 5y = 10$.

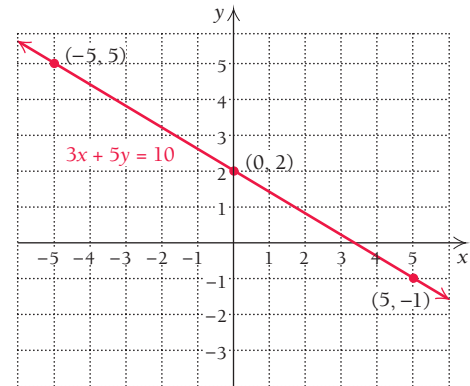
Solution We could choose x -values, substitute, and solve for y -values, but we first solve for y to ease the calculations.*

$$\begin{aligned}
 3x + 5y &= 10 \\
 3x + 5y - 3x &= 10 - 3x && \text{Subtracting } 3x \text{ from both sides} \\
 5y &= 10 - 3x && \text{Simplifying} \\
 \frac{1}{5} \cdot 5y &= \frac{1}{5} \cdot (10 - 3x) && \text{Multiplying both sides by } \frac{1}{5}, \text{ or dividing} \\
 &&& \text{both sides by 5} \\
 y &= \frac{1}{5} \cdot (10) - \frac{1}{5} \cdot (3x) && \text{Using the distributive law} \\
 &= 2 - \frac{3}{5}x && \text{Simplifying} \\
 &= -\frac{3}{5}x + 2
 \end{aligned}$$

Next we use $y = -\frac{3}{5}x + 2$ to find some ordered pairs, choosing multiples of 5 for x to avoid fractions.

*Be sure to consult Appendix A, as needed, for a review of algebra.

x	y	(x, y)
0	2	(0, 2)
5	-1	(5, -1)
-5	5	(-5, 5)

**Quick Check 2** ✓Graph: $3x - 5y = 10$.

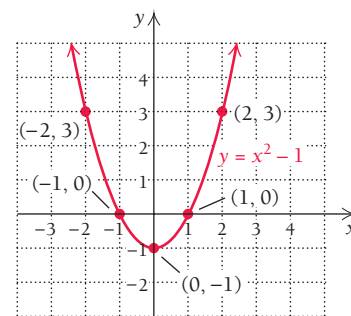
We plot the points, draw the line, and label the graph as shown.

2 ✓

Examples 1 and 2 show graphs of linear equations. Such graphs are considered in greater detail in Section R.4.

EXAMPLE 3 Graph: $y = x^2 - 1$.**Solution**

x	y	(x, y)
-2	3	(-2, 3)
-1	0	(-1, 0)
0	-1	(0, -1)
1	0	(1, 0)
2	3	(2, 3)

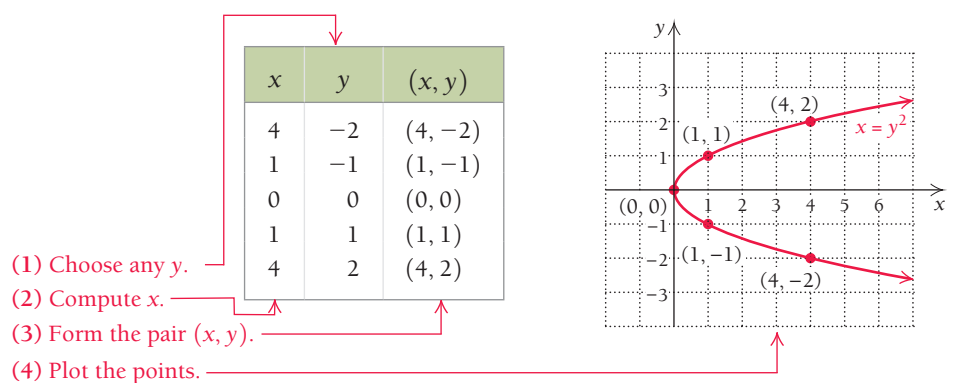
**Quick Check 3** ✓Graph: $y = 2 - x^2$.

This time the pattern of the points is a curve called a *parabola*. We plot enough points to see a pattern and draw the graph.

3 ✓

EXAMPLE 4 Graph: $x = y^2$.

Solution In this case, x is expressed in terms of the variable y . Thus, we first choose numbers for y and then compute x .

(1) Choose any y .(2) Compute x .(3) Form the pair (x, y) .

(4) Plot the points.

Quick Check 4 ✓Graph: $x = 1 + y^2$.

We plot these points, keeping in mind that x is still the first coordinate and y the second. We look for a pattern and complete the graph by connecting the points.

4 ✓

TECHNOLOGY CONNECTION

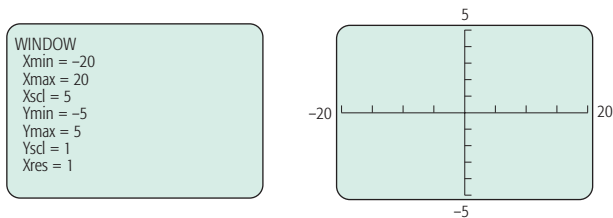


Introduction to the Use of a Graphing Calculator: Windows and Graphs

Viewing Windows

In this first of the optional Technology Connections, we begin to create graphs using a graphing calculator. Most of the coverage will refer to the various models of TI-83 and TI-84 calculators but in a somewhat generic manner, discussing features common to most graphing calculators. Although some keystrokes will be listed, exact keystrokes can be found in the owner's manual for your calculator or in the *Graphing Calculator Manual (GCM)* that accompanies this text.

The **viewing window** is a feature common to all graphing calculators. This is the rectangular screen in which a graph appears. Windows are described by four numbers in the format [L, R, B, T], representing the Left and Right endpoints of the x -axis and the Bottom and Top endpoints of the y -axis. A WINDOW feature can be used to set these dimensions. Below is a window setting of $[-20, 20, -5, 5]$ with axis scaling denoted as $Xscl = 5$ and $Yscl = 1$, which means that there are 5 units between tick marks extending from -20 to 20 on the x -axis and 1 unit between tick marks extending from -5 to 5 on the y -axis.

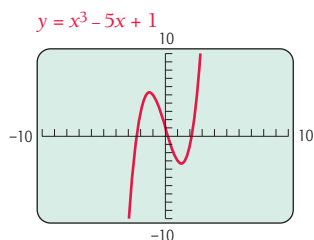


Scales should be chosen with care, since tick marks become blurred and indistinguishable when too many appear. On most graphing calculators, a window setting of $[-10, 10, -10, 10]$, $Xscl = 1$, $Yscl = 1$, $Xres = 1$ is considered **standard**.

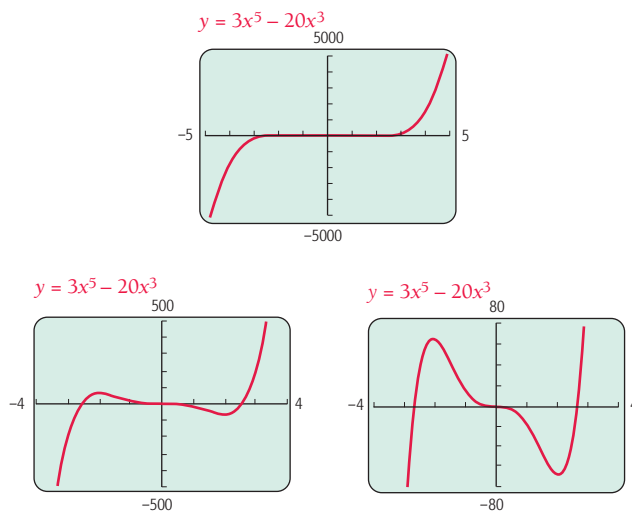
Graphs are made up of black rectangular dots called **pixels**. The setting $xres$ allows users to set pixel resolution at 1 through 8 for graphs of equations. At $Xres = 1$, equations are evaluated and graphed at each pixel on the x -axis. At $Xres = 8$, equations are evaluated and graphed at every eighth pixel on the x -axis. The resolution is better for smaller $Xres$ values than for larger ones.

Graphs

Let's use a graphing calculator to graph the equation $y = x^3 - 5x + 1$. The equation can be entered by pressing Y= and entering $x^3 - 5x + 1$. We obtain the following graph in the standard viewing window.



It is often necessary to change viewing windows in order to best reveal the curvature of a graph. For example, each of the following is a graph of $y = 3x^5 - 20x^3$, but with a different viewing window. Which do you think best displays the curvature of the graph?

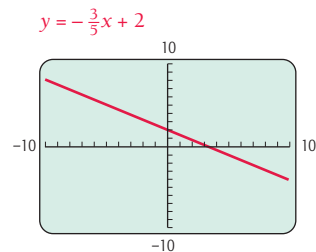


In general, choosing a window that best reveals a graph's characteristics involves some trial and error and, in some cases, some knowledge about the shape of the graph.

To graph an equation like $3x + 5y = 10$, most calculators require that the equation be solved for y . Thus, we must rewrite the equation and enter it as

$$y = \frac{-3x + 10}{5}, \quad \text{or} \quad y = -\frac{3}{5}x + 2.$$

(See Example 2.) Its graph is shown below.



To graph an equation like $x = y^2$, we solve for y and get $y = \sqrt{x}$ or $y = -\sqrt{x}$. We then graph both equations, $y_1 = \sqrt{x}$ and $y_2 = -\sqrt{x}$.

EXERCISES

Graph each of the following equations. Select the standard window, $[-10, 10, -10, 10]$, with $Xscl = 1$ and $Yscl = 1$.

- $y = x + 3$
- $y = x - 5$
- $y = 2x - 1$
- $y = 3x + 1$
- $y = -\frac{2}{3}x + 4$
- $y = -\frac{4}{5}x + 3$
- $2x - 3y = 18$
- $5y + 3x = 4$

(continued)

9. $y = x^2$ 10. $y = (x + 4)^2$ 17. $y = |x|$ (On most calculators, this is entered as $y = \text{abs}(x)$.)
11. $y = 8 - x^2$ 12. $y = 4 - 3x - x^2$ 18. $y = |x - 5|$ 19. $y = |x| - 5$
13. $y + 10 = 5x^2 - 3x$ 14. $y - 2 = x^3$ 20. $y = 9 - |x|$
15. $y = x^3 - 7x - 2$ 16. $y = x^4 - 3x^2 + x$

Mathematical Models

When a real-world situation is described using mathematics, the description is a **mathematical model**. For example, the natural numbers constitute a mathematical model for situations in which counting is essential. Also, the speed at which a body falls due to gravity can be described using a mathematical model.

Mathematical models are abstracted from real-world situations. A mathematical model may give results that allow us to predict what will happen in the real-world situation. If the predictions are too inaccurate or the results of experimentation do not conform to the model, the model must be changed or discarded.

CREATING A MATHEMATICAL MODEL

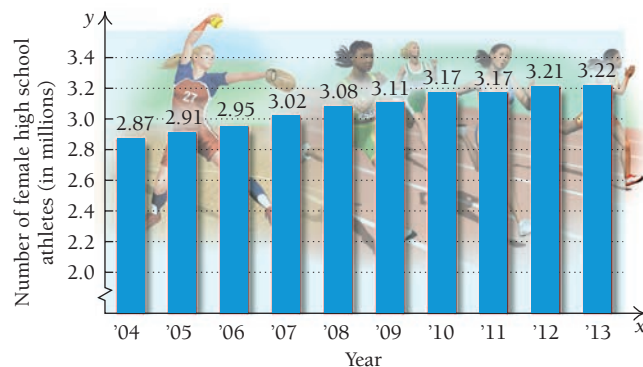
1. Recognize a real-world problem. →
2. Collect data. →
3. Analyze the data. →
4. Construct a model. →
5. Test and refine the model. →
6. Explain and predict.

Mathematical modeling is often an ongoing process. For example, finding a mathematical model that will provide an accurate prediction of population growth is not a simple task. Any population model that can be devised will need to be reshaped as further information is acquired.

EXAMPLE 5 The graph below shows participation by females in high school athletics from 2004 to 2013.



PARTICIPATION OF FEMALES IN HIGH SCHOOL ATHLETICS



(Source: National Federation of State High School Associations.)

Use the model $N = 0.041t + 2.88$, where t is the number of years after 2004 and N is the number of participants, in millions, to predict the number of female high school athletes in 2017.

Quick Check 5

Using the model in Example 5, determine the year in which the number of female high school athletes will reach 3.5 million.

Solution Since 2017 is 13 years after 2004, we substitute 13 for t :

$$N = 0.041t + 2.88 = 0.041(13) + 2.88 = 3.41.$$

According to this model, in 2017, approximately 3.41 million females will participate in high school athletics.

As is the case with many kinds of models, the model in Example 5 is not perfect. For example, for $t = 1$, we get $N = 2.921$, a number slightly different from the 2.91 in the original data. But, for purposes of estimating, the model is adequate. The cubic model $N = -0.00043t^3 + 0.003t^2 + 0.044t + 2.87$ also fits the data, at least in the short term: For $t = 1$, we get $N \approx 2.92$, close to the given data value. But for $t = 13$, we have $N = 3.00$, which is quite different from the prediction in Example 5. The difficulty with a cubic model here is that, eventually, its predictions become undependable. For example, the model in Example 5 predicts that there will be 3.54 million female high school athletes in 2020, but the cubic model predicts only 2.58 million. We always have to subject our models to careful scrutiny.

Compound Interest

One important model that is extremely precise involves **compound interest**. Suppose we invest P dollars at interest rate r , expressed as a decimal, and compounded annually. The amount A_1 in the account at the end of the first year, is given by

$$A_1 = P + Pr = P(1 + r). \quad \text{The original amount invested, } P, \text{ is called the principal.}$$

Going into the second year, we have $P(1 + r)$ dollars, so by the end of the second year, we will have the amount A_2 given by

$$A_2 = A_1 \cdot (1 + r) = [P(1 + r)](1 + r) = P(1 + r)^2.$$

Going into the third year, we have $P(1 + r)^2$ dollars, so by the end of the third year, we will have the amount A_3 given by

$$A_3 = A_2 \cdot (1 + r) = [P(1 + r)^2](1 + r) = P(1 + r)^3.$$

In general, we have the following theorem.

THEOREM 1

If an amount P is invested at interest rate r , expressed as a decimal, and compounded annually, in t years it will grow to the amount A given by

$$A = P(1 + r)^t.$$

EXAMPLE 6 Business: Compound Interest. Suppose \$1000 is invested in Fibonacci Investment Fund at 5%, compounded annually. How much is in the account at the end of 2 yr?

Solution We substitute 1000 for P , 0.05 for r , and 2 for t into the equation $A = P(1 + r)^t$ and get

$$\begin{aligned} A &= 1000(1 + 0.05)^2 && \text{Substituting} \\ &= 1000(1.05)^2 && \text{Adding terms in parentheses} \\ &= 1000(1.1025) && \text{Squaring} \\ &= \$1102.50. && \text{Multiplying} \end{aligned}$$

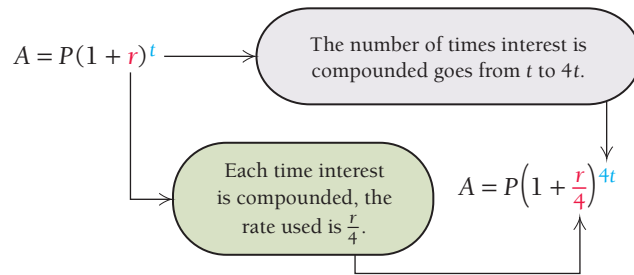
There is \$1102.50 in the account after 2 yr.

6 ✓

Quick Check 6 ✓

Business. Repeat Example 6 for an interest rate of 6%.

For interest that is compounded quarterly (four times per year), we can find a formula like the one above, as illustrated in the following diagram.



“Compounded quarterly” means that the interest is divided by 4 and compounded four times per year. In general, the following theorem applies.

THEOREM 2

If a principal P is invested at interest rate r , expressed as a decimal, and compounded n times a year, in t years it will grow to an amount A given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

EXAMPLE 7 Business: Compound Interest. Suppose \$1000 is invested in Wellington Investment Fund at 5%, compounded quarterly. How much is in the account at the end of 3 yr?

Solution We use the equation $A = P(1 + r/n)^{nt}$, substituting 1000 for P , 0.05 for r , 4 for n (compounding quarterly), and 3 for t . Then we get

$$\begin{aligned} A &= 1000\left(1 + \frac{0.05}{4}\right)^{4 \cdot 3} && \text{Substituting} \\ &= 1000(1 + 0.0125)^{12} \\ &= 1000(1.0125)^{12} \\ &= 1000(1.160754518) && \text{Using a calculator to approximate } (1.0125)^{12} \\ &= 1160.754518 \\ &\approx \$1160.75. && \text{The symbol } \approx \text{ means “approximately equal to.”} \end{aligned}$$

Quick Check 7 ✓

Business: Compound Interest.

Repeat Example 7 for an interest rate of 6%.

There is \$1160.75 in the account after 3 yr.

7 ✓

A calculator with a (y^x) or (\wedge) key and a ten-digit readout was used to find $(1.0125)^{12}$ in Example 7. The number of places on a calculator may affect the accuracy of the answer. Thus, you may occasionally find that your answers do not agree with those at the back of the book, which were found on a calculator with a ten-digit readout. In general, when using a calculator, do all computations and round only at the end, as in Example 7. Usually, your answer will agree to at least four digits. It is a good idea to consult with your instructor about the accuracy required.

Section Summary

- Most graphs can be created by plotting points and looking for patterns. A graphing calculator can create graphs rapidly.
- Mathematical equations can serve as models of many kinds of applications.
- An example of mathematical model is the formula for compound interest. If P dollars are invested at interest rate r , compounded n times a year for t years, then the amount A at the end of the t years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$